MATHEMATICAL & COMPUTATIONAL MODELING WITH APPLICATIONS A: STABILITY OF APPLIED DYNAMICAL SYSTEMS

(FALL 2021, UC RIVERSIDE)

Instructors: TBA (potential instructors: Mark Alber, Qixuan Wang, Jia Gou, Mykhailo Potomkin)
Lecture: TBA (3 hours per week)

Course description: The course intends to introduce students to the commonly used mathematical tools to study stability of solutions to evolution differential equations arising from various applications of interdisciplinary research. The course covers the stability of equilibria and limit cycles, as well as the modern theory of attractors. The focus of this course is on application of analytic and computational tools to investigate stability property of various models from Biology, Chemistry, and Physics.

Learning outcomes: After successful completion of this course, students are expected to be capable of identifying an appropriate tool from stability theory to analyze qualitative behavior of solutions for a given system of differential equations. In particular, students will learn how to classify type and stability of equilibria and Hopf bifurcation, as well as existence/nonexistence of global attractor. Moreover, students will be able to use computational tools for analysis of bifurcations.

Primary Textbook (not required but highly recommended)**:**

1. Strogatz, S.H., 2015. Nonlinear dynamics and chaos with student solutions manual: With applications to physics, biology, chemistry, and engineering. CRC press. 2nd edition.

Additional references:

2. Kuznetsov, Y.A., 2013 Elements of applied bifurcation theory (Vol. 112). Springer Science & Business Media.
3. Guckenheimer, J. and Holmes, P., 2013. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields (Vol. 42). Springer Science & Business Media, 7th printing of 1st edition of 1983.
4. Temam, R., 2012. Infinite-dimensional dynamical systems in mechanics and physics (Vol. 68). Springer Science & Business Media. 1st edition.
5. Wiggins, S., 2003. Introduction to applied nonlinear dynamical systems and chaos (Vol. 2). Springer Science & Business Media. 2nd edition.
6. Chueshov I., 2002. Introduction to the theory of infinite-dimensional dissipative systems. " Acta" publishers. 1st edition

Grading policy:

Letter grade, based on 5 written homework assignments and extra reading assignments (80%) and the final exam (20%)

- Written homework will be assigned each 2 weeks. Problems will be taken from the primary textbook and designed by the instructor depending on the topic (4 problems per homework).

- Extra reading assignments will be given every week to read specific chapters in the primary textbook including theories and examples. The reading assignments will be evaluated by giving problems related to the materials in the written homework.
- The final exam (approx. 5 questions) will be held in class and will be cumulative.

List of topics (10 weeks - two 80-minute lectures every week):

Week 1: (Textbook reference: [1, Chapters 2.3, 2.5]; [2, Chapter 1.4]; [3, Chapter 1.1])
Differential equations models. Examples: undamped/damped pendulum, population dynamics, epidemiological models, chemical kinetics, etc. Existence, uniqueness, and continuity of solutions.

Week 2: (Textbook reference: [1, Chapters 2.8, 3.6, 4.1])
Phase plane analysis. Large-time dynamics: analytical solutions vs geometric approach. Flow and bifurcations on the line, circle, and plane. Hysteresis phenomena. Potential, conservative, and reversible systems. Modern computational tools for bifurcation analysis.

Week 3: (Textbook reference: [1, Chapters 5.1, 5.2, 6.3]; [2, Chapters 1.3, 2.2])
Finding equilibria and analyzing their stability. Asymptotic stability vs Lyapunov stability. Linear 2x2 systems: full classification of equilibria. Almost linear system, linearization.

Week 4: (Textbook reference: [5, Chapters 1.2, 2])
Continuation of the stability of equilibria. Linear multi-dimensional systems. Routh-Hurwitz stability criterion. Method of Lyapunov function.

Week 5: (Textbook reference: [1, Chapters 7])
Stability of limit cycles. Poincare map and stability of limit cycles. Poincare-Bendixson Theorem. Relaxation oscillation including examples of Brusselator and chemical clock. Auto-oscillations.

Week 6: (Textbook reference: [1, Chapter 8.2]; [2, Chapter 3.4])
Continuation of Stability of limit cycles. Hopf bifurcations and their normal forms. Reduction of system exhibiting Hopf bifurcation to complex form and then to normal forms for 2D problems. Hopf bifurcations in higher dimensions.

Week 7: (Textbook reference: [4, Chapter 1.1]; [6, Chapters 1.2, 1.3, & 1.5])
Stability of invariant sets and global attractors. ω- and α-limit sets, dissipative systems and absorbing set, asymptotically compact and asymptotically smooth dynamical systems. Theorems on existence of global attractors.

Week 8: (Textbook reference: [4, Chapters 1.2]; [6, Chapter 4.5])
Continuation of the stability of invariant sets and global attractors. Examples: Duffing equation, Lorentz system, Minea system, simplified von Karman plate equation.

Week 9: (Textbook reference: [4, Chapter 7B.4]; [6, Chapter 1.6])
Conclusion of the stability of invariant sets and global attractors. LaSalle principle and structure of global attractors of gradient systems. Finite dimensionality of a global attractor.

Week 10: (Textbook reference: [6, Chapter 1.8])

Infinite dimensional system. Dimension reduction methods. Finite dimensional and exponential attractors. Idea of inertial manifolds.

MATHEMATICAL & COMPUTATIONAL MODELING WITH APPLICATIONS B: MULTI-SCALE MODELS WITH APPLICATIONS

(WINTER 2022, UC RIVERSIDE)

Instructors: TBA (potential instructors: Mark Alber, Qixuan Wang, Jia Gou, Mykhailo Potomkin)
Lecture: TBA (3 hours per week)

Course description: This example-based course introduces students to mathematical methods for problems with multiple separated scales, ubiquitous in interdisciplinary research. Such problems include those with small or large parameters which can be treated as 0 or infinity, respectively, but this treatment should be carried out carefully not to lose accuracy. This course covers boundary/interior layers in boundary value problems and homogenization theory, as well as other topics on multi-scale problems such as averaging in porous media and macroscopic limits in many particle systems.

Learning outcomes: After successful completion of this course, students are expected to be capable of identifying a small/large parameter in a given multi-scale problem and choosing an appropriate mathematical strategy for the complexity reduction.

Primary textbook:

1. Holmes, M.H., 2012. Introduction to perturbation methods (Vol. 20). Springer Science & Business Media. 1st edition.

Additional references:

2. A.N. Tikhonov, A.B. Vasil'eva, A.G. Sveshnikov, 1985. Differential Equations. Springer. 1st edition.
3. Lecture notes (UC Davis): https://www.math.ucdavis.edu/~hunter/asymptotics/asymptotics.html
4. Berlyand, L. and Rybalko, V., 2018. Getting Acquainted with Homogenization and Multiscale. Springer International Publishing. 1st edition.

Grading policy:

Letter grade, based on 5 written homework assignments and reading assignments (80%) and the final exam (20%)

- Written homework will be assigned each 2 weeks. Problems will be taken from the primary textbook and designed by the instructor depending on the topic (4 problems per homework).
- Extra reading assignments will be given weekly to read specific chapters in the primary textbook including theories and examples. The reading assignments will be evaluated by giving problems related to the materials in the written homework.
- The final exam (approx. 5 questions) will be held in class and will be cumulative.

List of topics (10 weeks - two 80-minute lectures every week):

Week 1: (Textbook reference: [1, Chapter 1]; [2, Chapter 7.2])

Concept of scale. Examples of multiscale problems. Regular and singular perturbations. Order symbols. First example: reaction kinetics and Tikhonov's theorem.

Week 2: (Textbook reference: [1, Chapter 2.2])
Matched Asymptotic Expansions – 1. 1D boundary value problems. Outer solutions, boundary layers, matching, composite expansions.

Week 3: (Textbook reference: [1, Chapter 2.3, 2.5, 2.6])
Matched Asymptotic Expansions – 2. 1D boundary value problems with interior and corner layers. Location of the layer

Week 4: (Textbook reference: [1, Chapter 2.7])
Matched Asymptotic Expansions – 2. Introduction to boundary layers in PDEs. Boundary-layer expansion and parabolic boundary layer in elliptic expansion.

Week 5: (Textbook reference: [4, Chapter 3], [3])
Homogenization theory. Effective conductivity (Maxwell formula) and effective viscosity (Einstein formula) in dilute systems. Introduction to two-scale expansion.

Week 6: (Textbook reference: [1, Chapter 5], [3])
Case study effective conductivity problem. Derivation of two-scale expansion and corrector problem. Example in Materials Science: checkerboard structure.

Week 7: (Textbook reference: [4, Chapter 11])
One-dimensional mass-spring system. Atomistic-to-continuum limit. Idea of Γ-convergence. Averaging for convex and non-convex elastic energies. Multi-dimensional atomistic-to-continuum limits: the Cauchy-Born rule.

Week 8: (Textbook reference: [1, Chapter 5.4])
The flow of a viscous fluid through a porous solid. Reynold's transport theorem. Reduction of porous flow using Homogenization. Homogenized problem: the Darcy's Law.

Week 9: (no textbook reference – please use course lecture notes)
Multi-phase models and sharp interface limits. Allen-Cahn equation for phase separation in multi-component alloy systems and derivation of mean curvature flow.

Week 10: (no textbook reference – please use course lecture notes)
Kinetic approach to many particle systems. Propagation of chaos and mean-field approximation. Empirical measure, the derivation of Vlasov equation. Thermodynamic limit.

MATHEMATICAL & COMPUTATIONAL MODELING WITH APPLICATIONS C: MODELING WITH DIFFERENTIAL EQUATIONS

(UC RIVERSIDE, SPRING 2022)

Instructors: TBA (potential instructors: Mark Alber, Qixuan Wang, Jia Gou, Mykhailo Potomkin)
Lecture: TBA (3 hours per week)

Course description: This course introduces students to mathematical modeling. Focus is on modeling with ordinary and partial differential equations with applications in Biology, Chemistry and Engineering.

Learning outcomes: After successful completion of this course, students are expected to be capable of designing a differential equation model to describe a given phenomenon studied in interdisciplinary research.

Recommended sources (a textbook is not required in this course)**:**

1. De Vries, G., Hillen, T., Lewis, M., Müller, J. and Schönfisch, B., 2006. A course in mathematical biology: quantitative modeling with mathematical and computational methods. Society for Industrial and Applied Mathematics, 1st edition.
2. Murray, J.D., 2007. Mathematical biology: I. An introduction (Vol. 17). Springer Science & Business Media, 3rd edition.
3. Edelstein-Keshet, L., 2005. Mathematical models in biology. Society for Industrial and Applied Mathematics, 1st edition.
4. Mase, G.E., 1970. Theory and problems of continuum mechanics: Schaum's Outline Series. McGraw-Hill, 1st edition.

Grading policy:
Letter grade, based on three individual homework assignments involving extra reading assignments (60%) and the final group project (40%)

- Three written homework assignments will be assigned. Each assignment will involve one or two extra reading assignments. Extra reading assignment will be assigned biweekly to read specific papers suggested by the instructor or chapters from the recommended reading materials. The extra reading assignments will be evaluated by giving problems related to the materials in the written homework.
- Final projects will be based on recent scientific papers related to topics covered in the course. The list of the papers will be given to students during the first week of classes. In this project, the students' task is to describe the model from the chosen paper in a 20-minute in-class presentation.

List of topics (10 weeks - two 80-minute lectures every week):
Week 1: (Textbook reference: [1, Chapter 3.2]; [2, Chapter 6.1])
Modeling with ordinary differential equations – 1. Introduction of basic reaction kinetics: the law of mass action, enzyme kinetics, Michaelis-Menton quasi-steady state analysis, activation and inhibition. Nondimensionalization.

Week 2: (Textbook reference: [1, Chapter 3.3]; [2, Chapter 7])
Modeling with ordinary differential equations – 2. Introduction of models of biological oscillators, Belousov reactions, Hodgkin-Huxley theory. Qualitative analysis of these models such as the existence of steady state, limit cycles, and bifurcations.

Week 3: (Textbook reference: [3, Chapter 6])
Modeling with ordinary differential equations – 3. Introduction to population models: logistic-type models, competition models, predator-prey models, epidemic models. Parameter estimation and sensitivity analysis.

Week 4: (Textbook reference: [2, Chapters 11.1, 11.2])
Modeling with partial differential equations – 1. Partial derivatives. An aged-structured model, renewal theorem. Diffusion. Relation between discrete and continuous description of diffusion. Random walk on a one-dimensional uniform lattice and its PDE continuous limit.

Week 5: (Textbook reference: [2, Chapters 11.3, 11.4])
Modeling with partial differential equations – 2. Models of chemotaxis in bacterial colonies. Keller-Segel model of chemotaxis. Models of dispersal in biological systems: from telegraph equations to model of locust population.

Week 6: (Textbook reference: [4, Chapters 2])
Modeling with partial differential equations – 3. Crash course to continuum mechanics. Strain and strain rate tensor. Cauchy stress tensor: Cauchy tetrahedron, normal and shear stresses, principal stress, and stress invariants. Force and torque balances with Cauchy stress tensor. Examples: biopolymer mechanics and thermal forces

Week 7: (Textbook reference: [4, Chapters 6.1, 7.1, 8.1, 9.1])
Modeling with partial differential equations – 4. Constitutive relations in continuum mechanics: elastic solids, fluid mechanics, and viscoelastic materials (Maxwell and Kelvin-Voight). Examples: swimming model, active gels, and tissue growth.

Week 8: (Textbook reference: [1, Chapter 6])
Introduction to cellular automata. The Game of Life cellular automata. Cellular Potts model. Cellular automata and nonlinear diffusion PDE model. Generalized cellular automata.

Weeks 9:
Specific examples with applications: **[*Instructors can pick 1-2 topics from the following list]**

(a) Boolean network models
(b) Cell and tissue growth model: on- and off-lattice models (agent-based, center based, vortex model, sub-cellular element model)
(c) Pattern formation: Turing patterns
(d) Kuramoto model of synchronization

Week 10:
Students' presentations.