MATHEMATICAL & COMPUTATIONAL MODELING WITH APPLICATIONS A: STABILITY OF APPLIED DYNAMICAL SYSTEMS  
(FALL 2021, UC RIVERSIDE)

Instructors: TBA (potential instructors: Mark Alber, Qixuan Wang, Jia Gou, Mykhailo Potomkin)  
Lecture: TBA (3 hours per week)

Course description: The course intends to introduce students to the commonly used mathematical tools to study stability of solutions to evolution differential equations arising from various applications of interdisciplinary research. The course covers the stability of equilibria and limit cycles, as well as the modern theory of attractors. The focus of this course is on application of analytic and computational tools to investigate stability property of various models from Biology, Chemistry, and Physics.

Learning outcomes: After successful completion of this course, students are expected to be capable of identifying an appropriate tool from stability theory to analyze qualitative behavior of solutions for a given system of differential equations. In particular, students will learn how to classify type and stability of equilibria and Hopf bifurcation, as well as existence/nonexistence of global attractor. Moreover, students will be able to use computational tools for analysis of bifurcations.

Primary Textbook (not required but highly recommended):

Additional references:

Grading policy:  
Letter grade, based on 5 written homework assignments and extra reading assignments (80%) and the final exam (20%)  
• Written homework will be assigned each 2 weeks. Problems will be taken from the primary textbook and designed by the instructor depending on the topic (4 problems per homework).
• Extra reading assignments will be given every week to read specific chapters in the primary textbook including theories and examples. The reading assignments will be evaluated by giving problems related to the materials in the written homework.

• The final exam (approx. 5 questions) will be held in class and will be cumulative.

List of topics (10 weeks - two 80-minute lectures every week):

Week 1: (Textbook reference: [1, Chapters 2.3, 2.5]; [2, Chapter 1.4]; [3, Chapter 1.1])

Differential equations models. Examples: undamped/damped pendulum, population dynamics, epidemiological models, chemical kinetics, etc. Existence, uniqueness, and continuity of solutions.

Week 2: (Textbook reference: [1, Chapters 2.8, 3.6, 4.1])


Week 3: (Textbook reference: [1, Chapters 5.1, 5.2, 6.3]; [2, Chapters 1.3, 2.2])


Week 4: (Textbook reference: [5, Chapters 1.2, 2])


Week 5: (Textbook reference: [1, Chapters 7])


Week 6: (Textbook reference: [1, Chapter 8.2]; [2, Chapter 3.4])

Continuation of Stability of limit cycles. Hopf bifurcations and their normal forms. Reduction of system exhibiting Hopf bifurcation to complex form and then to normal forms for 2D problems. Hopf bifurcations in higher dimensions.

Week 7: (Textbook reference: [4, Chapter 1.1]; [6, Chapters 1.2, 1.3, & 1.5])

Stability of invariant sets and global attractors. \( \omega \)- and \( \alpha \)-limit sets, dissipative systems and absorbing set, asymptotically compact and asymptotically smooth dynamical systems. Theorems on existence of global attractors.

Week 8: (Textbook reference: [4, Chapters 1.2]; [6, Chapter 4.5])

Continuation of the stability of invariant sets and global attractors. Examples: Duffing equation, Lorenz system, Minea system, simplified von Karman plate equation.

Week 9: (Textbook reference: [4, Chapter 7B.4]; [6, Chapter 1.6])

Conclusion of the stability of invariant sets and global attractors. LaSalle principle and structure of global attractors of gradient systems. Finite dimensionality of a global attractor.

Week 10: (Textbook reference: [6, Chapter 1.8])
Instructors: TBA (potential instructors: Mark Alber, Qixuan Wang, Jia Gou, Mykhailo Potomkin)
Lecture: TBA (3 hours per week)

Course description: This example-based course introduces students to mathematical methods for problems with multiple separated scales, ubiquitous in interdisciplinary research. Such problems include those with small or large parameters which can be treated as 0 or infinity, respectively, but this treatment should be carried out carefully not to lose accuracy. This course covers boundary/interior layers in boundary value problems and homogenization theory, as well as other topics on multi-scale problems such as averaging in porous media and macroscopic limits in many particle systems.

Learning outcomes: After successful completion of this course, students are expected to be capable of identifying a small/large parameter in a given multi-scale problem and choosing an appropriate mathematical strategy for the complexity reduction.

Primary textbook:

Additional references:

Grading policy:
Letter grade, based on 5 written homework assignments and reading assignments (80%) and the final exam (20%)
- Written homework will be assigned each 2 weeks. Problems will be taken from the primary textbook and designed by the instructor depending on the topic (4 problems per homework).
- Extra reading assignments will be given weekly to read specific chapters in the primary textbook including theories and examples. The reading assignments will be evaluated by giving problems related to the materials in the written homework.
- The final exam (approx. 5 questions) will be held in class and will be cumulative.

List of topics (10 weeks - two 80-minute lectures every week):
Week 1: (Textbook reference: [1, Chapter 1]; [2, Chapter 7.2])
Concept of scale. Examples of multiscale problems. Regular and singular perturbations. Order symbols. First example: reaction kinetics and Tikhonov’s theorem.
**Week 2:** (Textbook reference: [1, Chapter 2.2])

**Week 3:** (Textbook reference: [1, Chapter 2.3, 2.5, 2.6])
*Matched Asymptotic Expansions – 2.* 1D boundary value problems with interior and corner layers. Location of the layer

**Week 4:** (Textbook reference: [1, Chapter 2.7])

**Week 5:** (Textbook reference: [4, Chapter 3], [3])
*Homogenization theory.* Effective conductivity (Maxwell formula) and effective viscosity (Einstein formula) in dilute systems. Introduction to two-scale expansion.

**Week 6:** (Textbook reference: [1, Chapter 5], [3])
*Case study effective conductivity problem.* Derivation of two-scale expansion and corrector problem. Example in Materials Science: checkerboard structure.

**Week 7:** (Textbook reference: [4, Chapter 11])

**Week 8:** (Textbook reference: [1, Chapter 5.4])

**Week 9:** (no textbook reference – please use course lecture notes)
*Multi-phase models and sharp interface limits.* Allen-Cahn equation for phase separation in multi-component alloy systems and derivation of mean curvature flow.

**Week 10:** (no textbook reference – please use course lecture notes)
*Kinetic approach to many particle systems.* Propagation of chaos and mean-field approximation. Empirical measure, the derivation of Vlasov equation. Thermodynamic limit.
Instructors: TBA (potential instructors: Mark Alber, Qixuan Wang, Jia Gou, Mykhailo Potomkin)
Lecture: TBA (3 hours per week)

Course description: This course introduces students to mathematical modeling. Focus is on modeling with ordinary and partial differential equations with applications in Biology, Chemistry and Engineering.

Learning outcomes: After successful completion of this course, students are expected to be capable of designing a differential equation model to describe a given phenomenon studied in interdisciplinary research.

Recommended sources (a textbook is not required in this course):


Grading policy:
Letter grade, based on three individual homework assignments involving extra reading assignments (60%) and the final group project (40%)

- Three written homework assignments will be assigned. Each assignment will involve one or two extra reading assignments. Extra reading assignment will be assigned biweekly to read specific papers suggested by the instructor or chapters from the recommended reading materials. The extra reading assignments will be evaluated by giving problems related to the materials in the written homework.
- Final projects will be based on recent scientific papers related to topics covered in the course. The list of the papers will be given to students during the first week of classes. In this project, the students’ task is to describe the model from the chosen paper in a 20-minute in-class presentation.

List of topics (10 weeks - two 80-minute lectures every week):
Week 1: (Textbook reference: [1, Chapter 3.2]; [2, Chapter 6.1])
Week 2: (Textbook reference: [1, Chapter 3.3]; [2, Chapter 7])

*Modeling with ordinary differential equations – 2.* Introduction of models of biological oscillators, Belousov reactions, Hodgkin-Huxley theory. Qualitative analysis of these models such as the existence of steady state, limit cycles, and bifurcations.

Week 3: (Textbook reference: [3, Chapter 6])


Week 4: (Textbook reference: [2, Chapters 11.1, 11.2])


Week 5: (Textbook reference: [2, Chapters 11.3, 11.4])


Week 6: (Textbook reference: [4, Chapters 2])


Week 7: (Textbook reference: [4, Chapters 6.1, 7.1, 8.1, 9.1])


Week 8: (Textbook reference: [1, Chapter 6])


Weeks 9:

*Specific examples with applications: [*Instructors can pick 1-2 topics from the following list]*

- (a) Boolean network models
- (b) Cell and tissue growth model: on- and off-lattice models (agent-based, center based, vortex model, sub-cellular element model)
- (c) Pattern formation: Turing patterns
- (d) Kuramoto model of synchronization

Week 10:

*Students’ presentations.*