

Syllabus for the qualifying examination in Applied Mathematics

- Theory of ordinary differential equations: Basic existence, uniqueness theorems; Picard iteration, Banach fixed point theorem; Stable manifold theorem.
- Laplace equation: Fundamental solution; Distributional solutions; Poisson kernel; Green's function; Mean-value property for harmonic functions.
- Heat equation: Porous media equation; Self-similar solutions; Energy methods; Fourier transform.
- Linear transport equation: Conservation laws; Weak solution; Shock discontinuity; Entropy solution.
- Wave Equation: Spherical means.
- Sobolev spaces: Approximation by smooth functions; Extensions and traces; Sobolev inequalities; Compactness; Difference quotients; Fourier transform methods.
- Second order elliptic equations: Existence of weak solutions; Regularity of weak solutions; Maximum principles; Eigenvalue problems.
- Second order parabolic equations: Galerkin approximations; Energy estimates; Regularity of weak solutions; Maximum principles; Harnack's inequality.

References

- [1] Lawrence C. Evans. *Partial differential equations*, volume 19 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, second edition, 2010.
- [2] Michael Renardy and Robert C. Rogers. *An introduction to partial differential equations*, volume 13 of *Texts in Applied Mathematics*. Springer-Verlag, New York, 1993.