REAL ANALYSIS: MATH 209

MATH 209A

Textbook. The textbook is Gerald Folland's *Real Analysis*.

Reference. A very useful reference is H. L. Royden's *Real Analysis*, or the 4th edition of this book written by Royden and P. Fitzpatrick.

We will cover approximately the following material:

- Preliminaries — Chapter 0
- Measures — Chapter 1
- Integration — Chapter 2

Topics include:

- Properties of both abstract and Lebesgue-Stieltjes measures
- Caratheodory extension process constructing a measure on a sigma-algebra from a premeasure on an algebra; construction of Lebesgue-Stieltjes measure via this process
- Borel measures; complete measures; sigma-finite measure spaces
- Properties of measurable functions
- Abstract integration as well as Lebesgue integration on $\mathbb{R}^n$
- Dominated and monotone convergence theorems, Fatou's Lemma
- Special examples: Cantor sets, Cantor function, construction of a non-Lebesgue measurable subset of [0, 1].
- Modes of convergence: pointwise, uniform, almost everywhere, in measure, in L^1-norm, and implications between modes of convergence; Egoroff's and Lusin's theorems
- Product measures: Fubini's theorem and Tonelli's theorem
- Relation of Lebesgue integral to Riemann integral

MATH 209B

Textbook. The textbook is Gerald Folland's *Real Analysis*.

Reference. A very useful reference is H. L. Royden's *Real Analysis*, or the 4th edition of this book written by Royden and P. Fitzpatrick.

We will cover approximately the following material:

- Signed Measures and Differentiation — Chapter 3
- Point Set Topology — Sections 4.1—4.7
- Normed Vector Spaces, Linear Functionals, and the Baire Category Theorem and its Consequences — Sections 5.1—5.3
- Topological vector spaces—Chapter 5.4

Topics include:

- Radon-Nikodym theorem; Hahn, Jordan, and Lebesgue decompositions
- Lebesgue's differentiation theorem in $\mathbb{R}^n$; functions of bounded variation, absolute continuity
- Nets, Urysohn's lemma, compactness, the Stone–Weierstrass theorem, product topologies, Tychonoff's theorem
- Normed vector spaces: Banach spaces, quotients, adjoints, Hahn-Banach Theorem, Baire category theorem, open mapping theorem, closed graph theorem, the uniform boundedness principle
- Topological vector spaces: weak topology, weak-$*$ topology, Alaoglu's theorem

MATH 209C

Textbook. The textbook is Gerald Folland's *Real Analysis*.

Reference. A very useful reference is H. L. Royden's *Real Analysis*, or the 4th edition of this book written by Royden and P. Fitzpatrick.

We will cover approximately the following material:

- Hilbert spaces — Section 5.5
- L^p spaces — Chapter 6
- The dual of $C_c(X)$ and $C_0(X)$ — Sections 7.1 and 7.3
- Fourier analysis — Chapter 8.1—8.3 and 8.7
- Distributions — Chapters 9.1 and 9.2

Topics include:

- Hilbert spaces: Cauchy-Schwarz inequality, parallelogram law, Pythagorean theorem, Bessel's inequality, Parseval's identity, Riesz representation theorem, orthonormal bases
- L^p and l^p spaces: Hölder and Minkowski inequalities, duals of these spaces

- Various classes of functions: C^∞ , C_c^∞, C_c , C_0 and their duals
- Fourier analysis on $\mathbb{T}^n$ and $\mathbb{R}^n$, convolution, Fourier inversion theorem, Young's and Hausdorff-Young inequalities, applications to partial differential equations
- Schwarz functions and tempered distributions, convolution of tempered distributions, the Fourier transform of tempered distributions