

**SPECIAL
MATHEMATICS
COLLOQUIUM**

Nathan Habegger

université de Nantes

*"On the work of Xiao-Song Lin;
from classical to quantum topology"*

Friday, May 9, 2008

1:10-2:00pm, tea time @ 12:30 in faculty Lounge

Surge 268

In 1954, John Milnor introduced the notion of link homotopy and his invariants of links which he used to classify 3 component links up to homotopy. In 1987 the speaker and XS Lin achieved the classification, for any number of components, essentially by refining the Milnor invariants.

The Habegger-Lin classification scheme was extended to other equivalence relations in Lin's thesis and to more general concordance-type relations satisfying a list of 6 axioms. Axioms 1-4 are local, axiom 5 says that any string link (or 'pure tangle' as in pure braid) has an inverse, while axiom 6 says the equivalence relation on links is generated by isotopy and the equivalence relation on string links (every string link yields a link after 'closure').

In the early 90's Birman and Lin studied the work of Vassiliev on links and described in simple terms the Vassiliev filtration. Bar-Natan adopted their description as a definition of 'finite type' invariants and eventually all this was tied back to the perturbative Chern Simons quantum invariants via the Kontsevich Integral.

Early on, Lin suggested the Milnor invariants were of finite type, but this is strictly true only of the string link invariants because Milnor's invariants are only 'partially' defined, i.e. their indeterminacy depends on the lower order invariants. The speaker and G. Masbaum actually gave in 1997 a formula computing the Milnor string link invariants from the Kontsevich Integral. The tree-like Feynman diagrams correspond to the Milnor invariants.

The nagging problem that Vassiliev invariants of links are universally defined, but Milnor invariants, which ultimately gave the link-homotopy classification, are only partially defined, suggests that finite-type invariants of links are deficient. It turns out that axiom 6 of the aforementioned classification scheme is not satisfied so that Vassiliev (finite type) invariants of links can and ought to be refined, as shown in a recent preprint by the speaker and JB Meilhan.