UNIVERSITY OF CALIFORNIA RIVERSIDE

DEPARTMENT OF MATHEMATICS

COLLOQUIUM

Alexander Turbiner

(Universidad Nacional Autonoma de Mexico)

"Solvable Schoedinger Equations and Representation Theory"

Abstract:

A notion of solvability of the eigenvalue problem for a Schroedinger operator is introduced. It is shown that a classification of known exact solutions is related to a classification of spaces of polynomials which are finite-dimensional representation spaces of certain Lie algebras of differential operators. As a result the Lie-algebraic theory of the exact solutions of differential (difference) equations occurs. As a byproduct a new procedure of calculation of the Selberg integrals emerges.

Exact solutions of non-trivial Schroedinger equations are crucially important for both theory and applications. Amlost unique source of these solutions is the Calogero-Moser-Sutherland (CMS) quantum integrable systems (rational and trigonometric) naturally emerging in the Harish-Chandra theory (the Hamiltonian Reduction Method) and their generalizations. A Lie-algebraic theory of these systems can be developed giving representation theory origin of exact solutions. It can be shown that all classical $A-B-C-D$ CMS Hamiltonians (both rational and trigonometric) come from a single quadratic polynomial in generators of the maximal affine subalgebra of the gl(n)-algebra but unusually realized. The memory about the $A-B-C-D$ origin is kept in coefficients of the polynomial. For the case of exceptional $E_{8,7,6}-F_4-G_2$ and non-crystallographic $H_3, H_4, I_2(k)$ CMS Hamiltonians unknown infinite-dimensional algebras admitting finite-dimensional irreps appear. Their eigenfunctions are classical orthogonal polynomials. They span an infinite set of ordered (nested) linear spaces forming an infinite flag which is invariant wrt specific (weighted)-projective transformations.
Such a consideration provides a new insight to a theory of orthogonal polynomials.

Lie-algebraic theory allows to construct the 'quasi-exactly-solvable'generalizations of the above Hamiltonians where a finite subset of eigenstates is known algebraically. A general notion of (quasi)-exactly-solvable spectral problem is introduced.

Wednesday, April 13th, 2011

Surge 284

4:10-5:00pm

Tea Time at 3:40pm