QUAL SYLLABI FOR MATH 201ABC

February 2010

The topics marked with an asterisk are considered undergraduate material and will be only briefly reviewed in the graduate sequence Math 201A-B-C.

Groups
1.* Basic properties of groups and homomorphisms
2.* Cosets and Lagrange's Theorem
3.* Normal subgroups, quotient groups, fundamental homomorphism theorems
4.* Symmetric groups, Cayley's theorem
5.* Alternating and dihedral groups
6. Groups acting on sets
7. Sylow's theorems
8. Universal properties of products and coproducts
9. Free groups, presentations of groups

Rings
1.* Ideals, quotient rings, homomorphism theorems
2. Fields, characteristic of a field, field of fractions of an integral domain
3. Prime and maximal ideals
4. Elementary properties of localization
5. Polynomial rings, ring of polynomials in one variable over a field is a principal ideal domain
6. Factorization in commutative rings
7. Principal ideal domains are unique factorization domains
8. Euclidean domains are principal ideal domains
9. Unique factorization in polynomial rings
10. Eisenstein's irreducibility criterion

Modules and linear algebra
1.* Basic properties of bases and dimension of vector spaces
2.* The relationship between matrices and linear transformations
3.* Inner products and orthogonal bases, Gram-Schmidt process
4.* Determinants, eigenvalues, Cayley-Hamilton Theorem
5. Submodules, quotient modules, homomorphism theorems
6. Direct sum, free modules
7. Exact sequences, Short Five Lemma
8. Projective modules and injective modules, any module is a quotient of a projective module and a submodule of an injective module
9. Hom, dual of a vector space, dual bases and maps
10. Tensor product, tensor, symmetric and exterior algebras
11. Structure of finitely generated modules over principal ideal domains, applications to abelian groups
12. Rational canonical forms and Jordan canonical forms

Fields

1.* Elementary properties of field extensions, degree of a finite extension
2. Existence and uniqueness of splitting fields
3. Existence and uniqueness of algebraic closure
5. Separable, normal and Galois extensions
6. Fundamental theorem of Galois theory
7. Galois groups of quadratic and cubic extensions
8. Finite fields and their Galois theory
9. Transcendence basis and transcendence degree

References:

T.W. Hungerford, Algebra
N. Jacobson, Basic Algebra I
S. Lang, Algebra
D. Dummit, R. Foote, Abstract Algebra
M. Atiyah, I. Macdonald, Commutative Algebra