Complex Analysis

Syllabus for the qualifying examination

1. Undergraduate material.

a) Complex numbers and their geometry
b) The Riemann-Stieltjes integral (chapter 6 in ref. 5)
c) Green's formula in two dimensions
d) Uniform convergence and equicontinuity of sequences of functions (chapter 7 in ref. 5); integrals of functions depending on parameters.

2. Elementary analytic functions and their mapping properties.

a) Linear fractional transformations and the Riemann sphere
b) Cross-ratio
c) The exponential and the logarithm
d) Trigonometric functions

3. The Cauchy-Riemann equations.

a) The operators ∂ and $\bar{\partial}$ in Cartesian and polar coordinates
b) The homogeneous equation $\bar{\partial}u = 0$; properties of $\mathrm{Re}(u)$ and $\mathrm{Im}(u)$.
c) The inhomogeneous equation $\bar{\partial}u = f$.

4. Cauchy theorem and its consequences.

a) Proofs and Cauchy's formula
b) Cauchy's inequalities
c) The uniqueness principle
d) The maximum modulus principle and Schwarz lemma
e) The open mapping principle
f) Liouville's theorem and the fundamental theorem of algebra
g) Winding numbers, the argument principle and Rouche's theorem.

5. Singularities of analytic functions.

a) Classification of singularities,
b) Casorati-Weierstrass theorem,
c) Residue theorem,
d) Computation of definite integrals.

6. Taylor and Laurent series.

a) Cauchy-Hadamard formula for the radius of convergence,

b) Abel's theorem,
c) Laurent series,
d) Infinite products,
e) The expansions of elementary functions in infinite series and in infinite products.

7. Conformal transformations.

a) Riemann's mapping theorem,
b) The reflection principle,
c) Elementary conformal transformations.

8. Harmonic functions.

a) Maximum principle,
b) Mean value theorem,
c) Poisson and Jensen formulae,
d) Dirichlet problem,
e) Subharmonic functions.

References.

1) L. Ahlfors, Complex Analysis, McGraw-Hill.

2) J. Conway, Functions of One Complex Variable, 2nd edition, Springer.

3) K. Knopp, Theory of Functions, Parts I and II; Problem Book, Vol. I and II, Dover.

4) R. Narasimhan, Complex Analysis in One Variable, Birkhäuser.

5) W. Rudin, Principles of Mathematical Analysis, 3d. edition, McGraw-Hill (undergraduate material, only chapters 6 and 7)

6) S. Saks and A. Zygmund, Analytic Functions, Warsaw.