Syllabus for the qualifying examination in Real Analysis

Undergraduate materials:

- $\mathbb{R}$ and $\mathbb{R}^n$
- Basic topology: compact and connected sets, convergent sequences, Cauchy sequences, metric space completion.
- Sequences and series – numerical
- Continuity
- Differentiation
- Riemann integral
- Sequences and series of function, uniform convergence
- Fourier series (Chapter 8 from [8])
- Several variables: differentiation, inverse and implicit function theorem, Stokes theorem
- Stone-Weierstrass theorem
- Arzela-Ascoli theorem

Measure Theory (209A)

- Properties of both abstract and Lebesgue-Stieltjes measures
- Caratheodory extension process constructing a measure on a sigma-algebra from a premeasure on an algebra; construction of Lebesgue-Stieltjes measure via this process
- Borel measures; complete measures; sigma-finite measure spaces
- Properties of measurable functions
- Abstract integration as well as Lebesgue integration on $\mathbb{R}^n$
- Dominated and monotone convergence theorems, Fatou's Lemma
- Special examples: Cantor sets, Cantor function, construction of a non-Lebesgue measurable subset of [0, 1].

- Modes of convergence: pointwise, uniform, almost everywhere, in measure, in L^1-norm, and implications between modes of convergence; Egoroff's and Lusin's theorems
- Product measures: Fubini-Tonelli's theorem
- Relation of Lebesgue integral to Riemann integral
- Radon-Nikodym theorem; Hahn, Jordan, and Lebesgue decompositions
- Lebesgue's differentiation theorem in $\mathbb{R}^n$; functions of bounded variation, absolute continuity

Functional analysis (209B)

- Normed vector spaces: Banach spaces, quotients, adjoints, Hahn-Banach Theorem, Baire Category theorem, open mapping theorem, closed graph theorem, the uniform boundedness principle.
- Topological vector spaces: weak topology, weak-$*$ topology, Alaoglu's theorem.
- Hilbert space: Schwartz' inequality, Parallelogram law, Pythagorean theorem, Bessel's inequality, Parseval's identity
- L^p spaces and l^p spaces: Holder, Minkowski inequalities, duals

Fourier analysis (209C)

- Various classes of functions: C^∞ , C_c^∞ , C_c , C_0. Schwarz class of functions and distributions, tempered distributions which are bounded linear functionals on the Schwarz class. Urysohn's lemma.
- Convolution, Fourier transform and its properties. Fourier inversion theorem, Young's, Hausdorff-Young inequalities.

References

[1] G. Folland, Real Analysis – Modern Techniques and Their Applications

[2] W. Rudin, Real and Complex Analysis

[3] H.Royden, Real Analysis

[4] E. Hewitt and K. Stromberg, Real and Abstract Analysis

[5] R.G. Bartle, A Modern Theory of Integration,

[6] M.M. Rao, Measure Theory and Integration, 2nd ed. 2004

[7] E.J. McShane, Unified Theory of Integration

[8] W. Rudin, Principles of Mathematical Analysis (undergraduate material)

[9] T. Apostol, Mathematical Analysis (undergraduate material)