

Math 140 – Polynomials and Number Systems

We shall construct a syllabus using the out-of-print book *The Algebraic Foundations of Mathematics* by Ross A. Beaumont and Richard S. Pierce supplemented by material from L. Childs, *A Concrete Introduction to Higher Algebra*.

Foundational material [1 week]

- 1-1 Sets [review]
- 1-3 The cardinal number of a set
- 1-4 The construction of sets from given sets [review]
- 1-5 The algebra of sets [review]
- 2-1 Proof by induction [review]
- 2-2 The binomial theorem
- 2-3 Generalizations of the induction principle
- 3-1 Definition of number [Covered in Math 144]
- 3-2 Operations with natural numbers [review]
- 3-3 The ordering of natural numbers [Covered in Math 144]

Basic number systems of mathematics [1 week]

- 4-1 Construction of the integers
- 4-2 Rings
- 4-3 Generalized sums and products
- 4-4 Integral domains
- 4-5 The ordering of integers
- 4-6 Properties of order

Elementary Number Theory [two weeks]

- 5-1 The division algorithm
- 5-2 Greatest common divisor
- 5-3 Fundamental theorem of arithmetic
- 5-5 Applications of the Fundamental theorem of arithmetic
[This material is covered in **Childs: Chapters 2,3,4**]

Congruences and congruence classes [.67 weeks]

Childs: Chapters 5, 6

Rational numbers [1 week]

- 6-1 Basic properties of rational numbers [review]
- 6-2 Fields
- 6-4 Equivalence relations
- 6-5 The construction of \mathbb{Q} . [Done as an ordered set in Math144]

Real numbers [1.33 weeks]

- 7-1 Development of the real numbers
- 7-2 The coordinate line
- Brief discussion of 7-3 Dedekind cuts and 7-4 Construction of the real numbers(no proofs)
- 7-6 Properties of complete ordered fields
- Review of 7-7 Infinite sequences and 7-8 Infinite series
- 7-9 Decimal representation

Polynomials [2 weeks]

- 9-1 Algebraic equations
- 9-2 Polynomials
- 9-3 The division algorithm for polynomials
- 9-4 Greatest common divisor in $F[x]$
- 9-5 The unique factorization theorem for polynomials
- 9-7 The roots of a polynomial
- 9-8 The fundamental theorem of algebra
- 9-9 The solution of third and fourth degree equations
- 9-12 Polynomials with rational coefficients
[This material is covered in **Childs: Chapters 14, 15 16**]

Finite Fields and error correcting codes [1 week]

Childs: Chapters 13, 29