Mathematics 137: Plane Curves

Syllabus

1. Introduction

Review classification of real affine conics as illustrating themes of *classification* and *equivalence* and the need for transition from real to complex and from affine to projective. (.2 WEEKS)

2. The projective line,

The projective line, its automorphisms and the matrix group they define. Homogeneous polynomials in 2 variables and their zero-sets; homogenization. The fundamental theorem of algebra and its homogneous analogue; 1-dimensional Study's Lemma. (.8 WEEKS)

3. Affine curves

The complex affine plane; affine conics. Affine curves in general: degree, tangents, singular points, irreducibility. Affine transformations and their group. Statement of Study's lemma (Nullstellensatz for plane curves). (2 WEEKS)

4. Projective curves

The projective plane (real and complex); homogeneous coordinates; projective duality. Projective conics and classification. Projective curves, homogeneous polynomials, tangents and Euler's theorem, affine forms. Projective transformations and equivalence. (2 WEEKS)

5. Algebraic tools

Basic ring-theoretic definitions. 1-variable polynomials over a ring, divisibility, division and GCD algorithms, factorization. The resultant. (2 WEEKS)

6. Study's Lemma

Proof of Study's Lemma. Weak Bezout theorem and applications such as Pascal's hexagon. (1 WEEK)

7. The intersection symbol

Axioms. Uniqueness. Proof of existence via resultants. Full Bezout theorem. Applications: Desargues theorem, rationality, more. (2 WEEKS)