MATHEMATICS 133

GEOMETRY

Text: *Euclidean and Non-Euclidean Geometry; An Analytical Approach*, by P. J. Ryan

This course treats the elementary theory of affine and projective planes, finite geometries, Euclidean and Non-Euclidean geometries, groups of transformations and other algebraic structures related to geometry. A listing of chapters in the text and summaries of the sections in the chapters are given below.

Ch. 0 Historical introduction

Approaches to the study of geometry, examples from empirical geometry, an overview of the course.

Ch. 1 Plane Euclidean geometry

The coordinate plane, vector algebra on $\mathbf{R}^2$, the Euclidean plane $\mathbf{E}^2$, lines and perpendicularity, parallelism, reflections, congruence and isometries, symmetry groups, translations and rotations.

Ch. 2 Affine transformations of the Euclidean plane

Affine transformations, the affine group, geometric behavior of affine transformations, rays and angles, symmetries, centroids and barycentric coordinates, angles and triangles, standard congruence theorems for angles and triangles, the angle sum of a triangle.

Ch. 3 Finite groups of isometries in $\mathbf{E}^3$

Cyclic and dihedral groups, conjugate subgroups, Leonardo's theorem, regular polygons and their symmetries and similarities.

Ch. 4 Geometry of the sphere

Background material from solid geometry and vector algebra, planes, incidence geometry of the sphere, the spherical triangle inequality, isometries of the sphere, Euler's formula, spherical triangles, congruence theorems and trigonometry, finite rotation groups and isometry groups of the sphere.

Ch. 5 The projective plane $\mathbf{P}^2$

Relation to the Euclidean plane, incidence properties of $\mathbf{P}^2$, homogeneous coordinates, theorems of Desargues and Pappus, the projective group, the Fundamental Theorem of Projective Geometry, geometric behavior of projective collineations, polarities.

Ch. 6 Distance geometry on $\mathbf{P}^2$

Distance and the triangle inequality, isometries and motions.

Ch. 7 The hyperbolic plane $\mathbf{H}^2$

Construction of $\mathbf{H}^2$, incidence geometry, perpendicularity, distance and isometries in $\mathbf{H}^2$, reflections, parallel displacements, angles and triangles, hyperbolic trigonometry, asymptotic triangles, quadrilaterals and regular polygons, congruence theorems, isometries of $\mathbf{H}^2$, circles and related Non-Euclidean curves.

(There are also six appendices on the axiomatic approach, sets and functions, groups, linear algebra, a proof of Theorem 2.2, and the trigonometric and hyperbolic functions.)