Mathematics 5A

Course Description

Title: The Principles of Calculus I.

Short Title: POC I.

Credit Statement: 5 Units.

Lecture: 3 hours per week, 2 meetings.

Discussion: 2 hours per week, 2 meetings.

Prerequisites: Placement exam.

Catalog Description: An introduction to the principles of decomposition, transformation, rigidity, and symmetry, and their application to the study of the elementary functions. Topics include the elementary functions and their applications, linear and rotational motion, reflection, scaling of the plane, degree of interaction, and algebraic definition of tangency.

Further Description: This course applies the principles of decomposition and transformation to the study of the rigidity and symmetry of elementary functions. The course replaces the standard pre-calculus course and introduces students to principle based learning and reasoning. Traditional topics from a precalculus course are covered in this course, but they are treated as consequences of concepts that are beyond the scope of traditional courses. Topics include: Set operations, linear and non-linear equations and systems of equations, basic operations with functions, piecewise functions, rigid motions of the plane, transforming functions, inversion of functions, elementary functions, sketching elementary functions, rigidity of polynomial and rational functions, periodicity, waves and traveling waves, exponential models of change, tangency in the polynomial and rational function setting as order two or greater intersections.

Primary Textbook: David Weisbart, Kinnari Atit, Bryan Carrillo, Dylan Noack, Cathy Lussier, and Yat Sun Poon; with the assistance of UCR XCITE. (In preparation). The Principles of Calculus. An e-Book commissioned by California Learning Lab (https://calearninglab.org) and obligated to be an Open Educational Resource.

Learning Outcomes. After successful completion of this course, students are expected to be able to perform the following:

I.1. Determine set containment, unions and intersections of sets, and set differences; identify sets using set-builder notation.

I.2. Recall and identify the notation for intervals; calculate solutions of systems of linear inequalities in one variable; sketch solutions to systems of inequalities as subsets of the line.

I.3. Recall the definition of a cartesian product and a relation; use the graphical representation of a function to determine its properties; compare the properties of two functions from graphical representations; create a graphical representation of a function with specified properties.

I.4. Recall and utilize the general form of linear and monomial functions; evaluate a function given by a formula; determine the range of some functions.

I.5. Recall the meaning of restriction to subdomains and the basic operations with functions (sum, product, quotient, composition); calculate the iterated sums, products, quotients, and composites formed by multiple functions; decompose a complicated function into simpler components in specified ways.

I.6. Recall the meaning of a refinement of a partition; computes the sum, product, and quotient of piecewise functions; determine the composite of two piecewise defined functions.

I.7. Evaluate real valued functions of several variables; determine the level sets of functions on a planar domain; sketch restrictions of a function of two variables.

I.8. Solve systems of linear equations by substitution or by elimination; sketch the feasible set of a single linear inequality and multiple linear inequalities in two variables; describe feasible sets with set-builder notation.

II.1. Identify the action of arrow on point in the line and in in the plane; identify coordinate representations for vectors; determine the action of vectors on a locus of points in the plane that satisfy a given equation.

II.2. Recognize the geometric meaning of scaling a vector; determine the polar form of a vector; determine equations for symmetric and asymmetric scalings of a locus of points in the plane.

II.3. Identify the fundamental units of a physical quantity; describe a physical quantity using different units; solve linear, simple nonlinear, and general nonlinear scaling problems; solve problems involving Galileo's square-cube law.

II.4. Recall the vector equation for a line; determine the position of a point on a line segment given information about distance or relative distance; determine the position of a particle that is at a point at one time and another point at another time, or that moves on a given line in a certain direction at a certain speed; use vectors to identify a point that divides a line segment into segments with specified ratios of lengths; formulate equations for piecewise linear motion.

II.5. Recall the expression for a vector that is perpendicular to another vector; calculate the position of a point on a line that is closest to a point in the plane; create rectangles in the plane with specified side lengths or ratios of side lengths; determine the equation for a locus of points in the plane that is the reflection of a locus of points that satisfy a given equation.

II.6. Recall the condition for invertibility of a function in a general setting; calculate the formula for the inverse of some simple functions; use restriction to make a non-invertible function invertible on a smaller domain.

II.7. Recall the group addition law for points on the unit circle in cartesian coordinates; recall the formula for rotation of points around the origin by a given angle; determine the equation for a locus of points in the plane that is the rotation about a given point of a locus of points that satisfy a given equation.

II.8. Recall the definition of a fraction of a circle; match the sine, cosine, and tangent functions with their geometric realizations as y-coordinate, x-coordinate, and slope; solve for all points on the unit circle with a single specified coordinate; calculate parameterizations of paths given by the motion of a particle on a circle with specified constant speed.

II.9. Interpret graphically rotation by half of a circle, reflection across the axes and the inversion of the y-axis by the map $y \mapsto \frac{1}{y}$ for all (x, y) with $y \neq 0$.

III.1. Recall that lines are determined by two points and planes are determined by three non-colinear points; identify parameterizations of spatial lines; identify equations for planes; recall the dot and cross products of two vectors; determine the intersection of lines and planes; parameterize the intersection of planes; identify planes by a normal vector and a point.

III.2. Sketch quadratic functions; derive the quadratic formula; recall the factor theorem; identify the remainder for polynomial division; identify the zeros and the orders of zeros of a factored polynomial function; identify the asymptotic behavior of a polynomial; sketch a polynomial function using local and global data; determine the solution to quadratic optimization problems.

III.3. Identify the zeros and polls (and their orders) of a rational function whose numerator and denominator are written in factored form; identify the asymptotic behavior of a rational function; use the local and global properties of a rational function in factored form to sketch the function; use the sketch of a rational function to determine its properties.

III.4. Identify commensurable partitions for piecewise rational functions; use graphical information to

determine the solutions to polynomial and rational inequalities; determine sums, products, quotients, and composites of piecewise rational functions.

IV.1. Identify the symmetries of various sets; recall the definition of an odd and even function; identify even and odd functions.

IV.2. Recall the definition of a period; recognize periodicity and existence of a fundamental periods from graphical information; identify the domains where trigonometric functions are invertible; sketch sinusoidal functions; calculate composites of trigonometric and inverse trigonometric functions; solve equations involving trigonometric functions; generate simulations of traveling waves and their superpositions; understand constructive and destructive interference for 1-D waves.

IV.3. Identify linear or exponential model of change; recall the properties of exponential functions and logarithms; solve simple exponential and logarithmic equations; formulate models of change given data; calculate half-line (or any "fractional life" or doubling life, etc.) of a function satisfying an exponential growth model; determine future values of a function that satisfies linear or exponential growth models given two values at different times.

IV.4. Determine the equations of lines that are tangent to polynomial and rational functions at given points; determine the reflections of lines off of planar curves.

IV.5. Recall the definition of tangency for roots; interpret the idea of tangency to an inverse function; determine the equations of lines that are tangent to certain inverse functions; identify points where a polynomial attains an extremal value; recall that tangential intersections are degree two intersections except for at most finitely many points.

Proposed Lecture Schedule:

Week	Sections	Topics
Review	**I.1–I.3**	I.1 Basic set theory; unions, intersections, set differences. I.2 Intervals and inqualities. I.3 Interpret graphical information.
1	**I.4–I.6**	I.4 Develop a first library of functions. I.5 Study operations of function. I.6 Piecewise functions.
2	**I.7, I.8**	I.7 Studying functions of several variables and their restrictions. I.8 Multiple linear inequalities.
3	**II.1–II.4**	II.1 Introduction to vectors and transformation. II.2 Scaling vectors. II.3 Physical units and scaling. II.4 Linear motion.
4	**II.5–II.8**	II.5 Orthogonality of vectors and reflection. II.6 Inverse functions. II.7 Rotation. II.8 Trigonometric functions; parameterization of rotational motion.
5	**II.9, III.1**	II.9 Reflection across axes; rotation by half of a circle; y-axis inversion. III.1 Spatial lines and planes.
6	**III.2, III.3**	(Mid-term) III.2 Sketching polynomial functions; factor theorem; quadratic optimization. III.3 Sketching rational functions.
7	**III.4, IV.1**	III.4 Rational inequalities, IV.1 Group of rigid motions.
8	**IV.1, IV.2**	IV.1 Introduction to symmetry; odd and even functions. IV.2 Periodic functions.

Week	Sections	Topics
9	**IV.2, IV.3**	IV.2 Trigonometric equation; traveling waves. IV.3 Properties of exponential functions and logarithms; exponential change.
10	**IV.4, IV.5**	IV.4 Tangency to polynomial and rational functions; tangency to circles and ellipses. IV.5 Tangency to inverse functions; detection of "wiggles"; higher order tangency.

Additional References:

1. Richard Courant and Fritz John (1999). Introduction to Calculus and Analysis, Volume 1, Reprint of the 1989 Edition. Springer-Verlag. DOI 10.1007/978-3-642-58604-0.
2. Michael Spivak (2008). Calculus, 4th edition, Publish or Perish. ISBN 978-0-914098-91-1.
3. James Stewart (2008). Calculus, 6th edition, ISBN-13: 978-0495011606
4. Claudia Neuhauser and Marcus Roper (2018). Calculus for Biology and Medicine, 4th edition, Pearson. ISBN-13: 978-0134122601.

Prepared by Yat Sun Poon and David Weisbart, Spring 2024.