Mathematics 5B

Course Description

Title: The Principles of Calculus II.

Short Title: POC II.

Credit Statement: 5 Units. Credit is awarded for only one of MATH 005B or MATH 007A or MATH009A or MATH09HA.

Lecture: 3 hours per week, 2 meetings.

Discussion: 2 hours per week, 2 meetings.

Prerequisites: MATH 005A with a grade of C- or better.

Catalog Description: An introduction to application of finite approximation to study functions of one variable. Topics include sequences, series, differential calculus, and antiderivatives.

Further Description: This course applies the principle of finite approximation to the study of the local linear approximation of functions. Topics include: finite approximation of planar area, sequences and their limits, analysis of error, continuous limits, continuity, asymptotic behavior, approximating rate of change, the derivative, approximation by the tangent line, derivatives of elementary functions, implicit differentiation, related rates, the geometry of particle motion, the mean value theorem, extremal points, the antiderivative, simple first order differential equations.

Primary Textbook: David Weisbart, Kinnari Atit, Bryan Carrillo, Dylan Noack, Cathy Lussier, and Yat Sun Poon; with the assistance of UCR XCITE. (In preparation). The Principles of Calculus. An e-Book commissioned by California Learning Lab (https://calearninglab.org) and obligated to be an Open Educational Resource.

Learning Outcomes. After successful completion of this course, students are expected to be able to perform the following:

V.1. Recall and understand the concept of area; calculate the area inside a simple closed polygonal curve.

V.2. Understand the concept of sequential limits; calculate sequential limits using limit laws, including the squeeze theorem; calculate sequential limits using a recursive formula; use error terms to compute more complicated sequential limits.

V.3. Recall and understand approximation methods for determining the area bounded by a circle; calculate sequential limits for trigonometric functions with respect to different angle measures.

V.4. Understand the definition of a sequence of partial sums; recognize the convergence of some common series; use the comparison test, alternating series test, limit comparison test, ratio and root tests.

V.5. Use limit laws and squeeze theorem to calculate limits of functions;

V.6. recall and understand the concept of continuity, the extreme value theorem, and the Intermediate Value Theorem; determine continuity at specific points and on specific intervals.

V.7. Describe error terms coming from differences of the form $f(x+h)-f(x)$; recall the concept of a modulus of continuity function; analyze error terms, including those arising from sums, products, and compositions of functions.

V.8. Recognize the relationship between the difference quotient and the average rate of change of a

function on an interval; calculate the limit of the difference quotient for a large collection of elementary functions.

V.9. Recall the concept of a partition for a function, a tagging, and the mesh of a partition; understand the definition of the Riemann integral and its interpretation in various contexts; use the Riemann integral to approximate or compute areas bounded by continuous functions on the plane.

VI.1. Understand the definition of the derivative of a function in terms of the local linear approximation of a function; recognize the connection between the limit of the difference quotient definition of the derivative and the definition in terms of local linear approximation; use local linearization to derive the rules of differentiation; approximate the value of a differentiable function near a known value.

VI.2. Calculate the derivative of the inverse of a differentiable function; appy the rules of differentiation, including the chain rule; calculate the derivative of implicitly defined functions.

VI.3. Recall Fermat's Theorem, Rolle's Theorem, and the Mean Value Theorem; identify antiderivatives; calculate the extremal values of a differentiable function; identify global extrema of a continuous and piecewise differentiable function; apply l'Hopital's rule to calculate limits of indeterminate forms.

VI.4. Recognize and analyze the following characteristics of a differentiable function: increasing vs decreasing, concavity, inflection; interpret these concepts in various contexts; sketch functions using zeroth, first, and second order information.

VI.5. Determine antiderivatives and relate antidifferentiation to solving simple first order differential equations.

VI.6. Calculate the velocity vectors associated to parametrized paths in the plane and in three dimensional space.

Proposed Lecture Schedule:

Week	Sections	Topics
Review	**V.1**	V.1 Area of polygons.
1	**V.2**	V.2 Sequences; sequential limits; the limit laws.
2	**V.3, V.4**	V.3 Measurement of a circle; limits involving trigonometric functions. V.4 Infinite series; convergence tests.
3	**V.5, V.6**	V.5 Limits; one-sided limits; infinite limits. V.6 Continuous functions; approximating continuous functions.
4	**V.7, V.8**	V.7 Analysis of error. V.8 Approximating change; exponential function.
5	**V.9**	V.9 Approximating area under a function; the Riemann integral.
6	**VI.1, VI.2**	(Mid-term) VI.1 Differentiation; differentiation rules. VI.2. Derivatives of inverse functions; implicit differentiation; related rates.
7	**VI.3**	VI.3 Extreme value theorem; mean value theorem; antiderivative and applications; l'Hopital's rule.
8	**VI.4**	VI.4 Concavity and curve sketching.
9	**VI.5**	VI.5 Antiderivatives and first order differential equations.
10	**VI.6**	VI.6 Particle motion in space.

Additional References:

1. Richard Courant and Fritz John (1999). Introduction to Calculus and Analysis, Volume 1, Reprint of the 1989 Edition. Springer-Verlag. DOI 10.1007/978-3-642-58604-0.
2. Michael Spivak (2008). Calculus, 4th edition, Publish or Perish. ISBN 978-0-914098-91-1.
3. James Stewart (2008). Calculus, 6th edition, ISBN-13: 978-0495011606
4. Claudia Neuhauser and Marcus Roper (2018). Calculus for Biology and Medicine, 4th edition, Pearson. ISBN-13: 978-0134122601.

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