MATH 163
Applied Dynamical Systems

Course Description
Introduction to dynamical systems with applications to problems in Physics, Biology, Chemistry, and Engineering. Topics include phase plane analysis, stability of dynamical systems, and numerical methods for dynamical systems.

Prerequisites
MATH 045 with a grade of C- or better or MATH 046 with a grade of C- or better; MATH 031 with a grade of C- or better; MATH 135A with a grade of C- or better; or equivalent; or consent of instructor.

Textbooks
Nonlinear Dynamics and Chaos: With Applications to Physics, *Biology, Chemistry, and Engineering* by Steven H. Strogatz

Dynamical Systems with Applications using MATLAB by Stephen Lynch

Mathematical Models in Biology by Leah Edelstein-Keshet

Suggested Lecture Schedule

Week #	Topic(s)
1, 2	1. Linear discrete dynamical systems. Analytic solution for the 1st and 2nd order. 2. Nonlinear discrete dynamical systems. Tent and logistic maps: fixed points, periodic orbits. Sensitivity to initial conditions and route to chaos. 3. Selected examples, including: a. Examples of discrete dynamical systems in Biology: Leslie model for population dynamics, and Epidemiological models (discrete SIR models). b. Examples of discrete dynamical systems in Medicine: Red Blood Cell Production model, Ventilation Volume and Blood CO2 Levels. c. Examples of discrete dynamical systems in Economics: dynamics of GDP and Irregular growth cycles, inflation-unemployment model. d. Introduction to Neural networks: iterative training and backpropagation algorithm (application to machine learning). Lab – Introduction to MATLAB, simulation epidemiological models, the model of red blood cell production, and inflation-unemployment model. Simple neural network (code will be provided).

3	1. From discrete to continuous dynamical systems, finite differences. 2. Existence & uniqueness of ODE solutions: when do solutions of an ODE form a dynamical system? 3. Examples of ODE-based dynamical systems: intracellular drug delivery, epidemiological model, population growth model. 4. Numerical solutions to ODEs: Euler methods. Lab – Numerical solution of ODEs using Euler methods in MATLAB
4, 5	1. Autonomous equations in 1D: stable, unstable, and semistable equilibria. 2. Linear system, nonlinear system, linearization and stability. 3. Phase portrait, typical bifurcations, and hysteresis. 4. Examples: Overdamped pendulum, Insect Outbreak model, Population growth model, Chemical kinetics, Epidemical model (reduced Kermack and McKendrick SIR model). Lab – Numerically simulate and analyze the pendulum equation and chemical Kinetics. Finding threshold parameter values for epidemic to occur in SIR model, calculating bifurcation curves in insect outbreak model.
6, 7	1. Fixed points: classification and linear stability. 2. Introduction to solutions of linear and almost linear 2x2 systems. 3. Example of 2D system: Lotka-Voltera, competing, cooperating species. 4. Example of 3D system: SIR model. 5. Selected bifurcations in 2D systems. 6. Limit cycles and nonlinear oscillations. Chemical kinetics (Brusselator and glycolytic oscillation). Supercritical and subcritical Hopf bifurcation. Lab – Bifurcation curves for SIR model with MATLAB. Hopf bifurcation values of parameters in model of competing species, using MATLAB.
8, 9	1. Introduction to probability space and random variables. 2. Stochastic differential equations and numerical integration (Euler-Maruyama scheme). 3. Examples: random Lotka-Voltera, Langevin dynamics for bacterial suspension model. Lab – Lattice model of cell tissue growth with random orientation. Langevin dynamics for interacting bacteria.

10	1. Interacting species: competing species & predator-prey. Source: [Lynch: Chapter 10]. Material: Dynamical System on planes, Linear analysis, and Limit cycles. 2. Oscillating chemical reactions: chemical kinetics, Brusselator, BZ reaction. Source: [Strogatz: Chapter 8.3]. Material: Limit cycles, Bifurcations in 2D. 3. Pendulum mechanical oscillations: undamped and damped, phase portrait, coupled pendula. Material: 2D dynamics, chaos in 3D. 4. Epidemiological models: SIR and others, coefficient R2. Source: [Ellner: Chapter 6]. 5. Neural Networks & Machine Learning. Source: [Lynch: Chapter 18]. 6. Biological oscillators and Synchronization: Kuramoto model. [wiki]