Math 045: Introduction to Ordinary Differential Equations for Physical Sciences and Engineering

Effective Quarter:	Winter 2022
Units:	4
Prerequisites:	MATH 007B or MATH 009B or MATH 09HB with a grade of C- or better
Title:	Introduction to Ordinary Differential Equations for Physical Sciences and Engineering
Short Title:	Intro to ODE for Phy Sci Engr
Credit Statement:	Credit is awarded for only one of MATH 045 or MATH 046; Cross-listed with EE 020A

Catalog Description: Introduction to ordinary differential equations; complex numbers; trigonometric, compact and exponential Fourier Series; frequency spectrum; Laplace transform, Fourier transform, and their application to the solution of integro-differential equations as they appear in the physical sciences and engineering.

Learning Outcomes: After successful completion of this course, students are expected to be able to perform the following:

- Recognize whether a first-order ODE is separable and, if so, solve it.
- Use Euler's method to find approximate solutions to a first-order ODE.
- Determine whether two real-valued functions are linearly independent.
- Solve a variety of second-order ODEs using the methods of undetermined coefficients and variation of parameters.
- Solve homogeneous and non-homogeneous second order ODEs with constant coefficients.
- Apply knowledge of ODEs to the solution of basic problems in the Physical Sciences and Engineering.
- Represent a complex number in either standard, polar, or exponential form, and perform basic algebraic operations with complex numbers.
- Compute the trigonometric ($\sin n\theta$, $\cos n\theta$) and complex exponential ($e^{in\theta}$) Fourier Series of periodic functions, and conduct a basic spectrum analysis.
- Express Fourier series in trigonometric, complex exponential, and "compact" ($\cos(n\omega t + \theta_n)$) forms and be able to transform between the forms.
- Solve ODEs with constant coefficients using the Fourier transform.
- Compute the Laplace and Fourier transforms of functions either by using the definition of the transforms or by using its properties such as linearity, scaling, time or frequency shifts, differentiation, and integration.
- Compute the inverse Laplace Transform for rational functions with simple, repeated and complex Poles.
- Solve ODEs with constant coefficients using the Laplace Transform.
- Relate the solution of ODEs to the particular physical application using complex numbers—this includes analyzing frequency spectrum by plotting its magnitude and phase or by making the Bode plot.

- Relate the Fourier transform to the Laplace transform of an imaginary argument.

Primary Textbook: *Elementary Differential Equations with Boundary Value Problems* by William F. Trench, a free text which can be found at https://digitalcommons.trinity.edu/mono/9/.

Additional References:

(1) Mary L. Boas, *Mathematical Methods in the Physical Sciences, Third Edition*, 2006 John Wiley & Sons
(2) K. F. Riley, M. O. Hobson, and S. J. Bence, *Mathematical Methods for Physics and Engineering, Third Edition*, 2006 Cambridge University Press.
(3) Merle C. Potter, Jack L. Lessing, and Edward F. Aboufadel, *Advanced Engineering Mathematics, Fourth Edition*, 2019 Springer Nature Switzerland.

Syllabus:

Week	**Sections**	**Topics**
1	1.1, 1.2, 2.1 2.2	Overview and linear equations Separable equations
2	2.6 3.1	Solving $y' + p(t)y = g(t)$ using an integrating factor Euler's method
3	– 5.1, 5.2	Initial introduction to complex numbers, up to an understanding of Euler's formula (introduce $e^{i\theta} = \cos\theta + i\sin\theta$) Homogeneous second order linear equations including concept of linear independence of functions and the Wronskian; constant coefficient case
4	5.3 5.4	Nonhomogeneous linear equations, and comparison of homogeneous, particular solutions to 0-forcing, 0-state solutions Method of undetermined coefficients (only the first of two sections)
5	5.7	Variation of parameters Review for midterm
6	6.1-6.3	Midterm Application of second order ODEs to spring problems and to the RLC Circuit
7	–	Further exploration of complex numbers, understanding them in polar or exponential form ($re^{i\theta}$), or magnitude, phase Fourier series—trigonometric, complex exponential ($e^{i\theta}$), and "compact" ($\cos(n\omega t + \theta_n)$)—and its application to ODE
8	– – 8.1, 8.2	Continuation of Fourier series, and the Fourier transform as a limit of Fourier series The Laplace transform and its inverse
9	8.2 8.3 8.4	Continuation of inverse Laplace transform Solving initial value problems with Laplace transform Laplace transform with step function as forcing
10	8.6	Using the convolution theorem to find the inverse Laplace transform of products Review for Final Exam

The *Sections* column refers to sections in the primary textbook (by Trench).