Math 213A: Numerical Methods for Partial Differential Equations I

UC Riverside Fall 2021

Instructor: TBA (possible instructors: Weitao Chen, Yat Tin Chow, Heyrim Cho),
Office: TBA, **Email**: TBA
Lectures: TBA (3 hours per week), **Room**: TBA, **Office Hour**: TBA

Primary Textbook:

- Partial Differential Equations with Numerical Methods by S. Larsson and V. Thomee, 2nd Edition, Springer

Suggested References:

- Finite Difference Schemes and Partial Differential Equations by J.C. Strikwerda, 2nd Edition, SIAM
- Numerical Solution of Partial Differential Equations: Finite Difference Methods by G.D. Smith, 3rd Edition, Oxford University Press

Course Description: This course is an introduction to the numerical methods for solving partial differential equations, especially parabolic and elliptic type equations. This class is fundamental for students who want to conduct researches in scientific computing and numerical methods for PDEs. It will teach students important and useful tools for solving PDE models on computer clusters with applications in interdisciplinary research such as fluid dynamics, mathematical biology, physics, engineering, chemistry, and biology. The course will cover different approaches for specific types of equations and study their properties including consistency, convergence and accuracy in the applications. The overall objective of this course is to provide students with the necessary mathematical knowledge and computational tools to understand and solve a variety of models in terms of partial differential equations.

Grading Criteria:

• Homework assignments (40%). Written homework problems, which will be assigned each week, are from the textbook. Homework problems include solving theoretical and practical problems inspired by the topics discussed in class. Extra reading assignments will be given weekly to read specific chapters including theories and examples in the primary textbook. The reading assignments will be evaluated by giving problems related to the materials in the written homework.

• Project assignments (40%). Computing projects will be assigned for each main topic, in addition to written homework, in which the numerical methods are studied in the context of a specific problem from scientific research. The computing projects will involve a combination of numerical simulation and mathematical analysis.

• Final examination (20%), given during the final exam time, closed book written exam with about 6 problems.

Tentative Course Schedule
This outline covers 10 weeks of two 80-minute lectures each week.

Week 1. Introduction to PDEs (Ch. 1-2)
Definitions of elliptic, parabolic, hyperbolic equations, norms, convergence,

Week 2. Introduction to Finite Difference Method (Ch. 3-4)

Week 3. Elliptic Equations (Ch. 5)
Finite element method

Week 4. Elliptic Equations (Ch. B)
Multigrid method

Week 5. Parabolic Equations (Ch. 8-9)
Explicit finite-difference approximation to parabolic equation

Week 6. Parabolic Equations (Ch. B)
Crank-Nicolson implicit method
Alternating direction implicit method

Week 7. Parabolic Equations (Ch. B)
Exponential time differencing method
Integration factor method

Week 8. Parabolic Equations (Ch. 14)
Spectral method

Week 9. Convection-diffusion Equation (selected papers)
Operator splitting method
Transformation into a fix domain
Level set approaches

Week 10. Nonlinear Parabolic Equations (Ch. A)
Convergence, stability and consistency
Lax's equivalence theorem

Math 213B: Numerical Methods for Partial Differential Equations II
UC Riverside Winter 2022

Instructor: TBA (possible instructors: Weitao Chen, Yat Tin Chow, Heyrim Cho),
Office: TBA, **Email**: TBA
Lectures: TBA (3 hours per week), **Room**: TBA, **Office Hour**: TBA

Textbook: Numerical Methods for Conservation Laws by Randall J. LeVeque. 2nd Edition, Birkhäuser

Course Description: This course is an introduction to the numerical methods for solving partial differential equations, especially hyperbolic conservation laws. This class is fundamental for students who want to conduct researches in scientific computing and numerical methods for PDEs. It will teach students important and useful tools for solving PDE models on computer clusters with applications in interdisciplinary research such as fluid dynamics, mathematical biology, physics, engineering, chemistry, and biology. The course will cover different approaches for specific types of equations and study their properties including consistency, convergence and accuracy in the applications. The overall objective of this course is to provide students with the necessary mathematical knowledge and computational tools to understand and solve a variety of models in terms of partial differential equations.

Grading Criteria:
• Homework assignments (40%). Written homework problems, which will be assigned each week, are from the textbook. Homework problems include solving theoretical and practical problems inspired by the topics discussed in class. Extra reading assignments will be given weekly to read specific chapters including theories and examples in the primary textbook. The reading assignments will be evaluated by giving problems related to the materials in the written homework.
• Project assignments (40%). Computing projects will be assigned for each main topic, in addition to written homework, in which the numerical methods are studied in the context of a specific problem from scientific research. The computing projects will involve a combination of numerical simulation and mathematical analysis.
• Final examination (20%), given during the final exam time, closed book written exam with about 6 problems.

Tentative Course Schedule
This outline covers 10 weeks of two 80-minute lectures each week.

Week 1. Introduction to Hyperbolic Conservation Laws (Ch. 1-2)
Applications, derivation of integral and differential formulations,
global error and convergence, local truncation error, stability, CFL condition

Week 2. Scalar Conservation Laws (Ch. 3)
Linear equation, Burgers' equation, shock formation, weak solutions

Week 3. Scalar Conservation Laws (Ch. 3-4)
Riemann problem, shock speed, entropy conditions
Some examples: traffic flow, two phase flow

Week 4. Numerical Methods for Linear Equations (Ch. 10)
Global error estimate and convergence, local truncation error estimate,
stability, Lax equivalence theorem, CFL condition,
upwind method

Week 5. Numerical Methods for Computing Discontinuities (Ch. 11)
First order methods with viscosity
Second order methods and dispersion

Week 6. Conservative Methods for Nonlinear Problems (Ch. 12)
Conservative methods, consistency, discrete conservation,
Lax-Wendroff Theorem, entropy condition

Week 7. Linear Systems (Ch. 6)
Characteristic variables, simple waves
Linearization of nonlinear systems, Riemann Problem

Week 8. Shocks and Rarefaction Waves for Systems (Ch. 7-8)
Hugoniot locus, solutions of the Riemann Problem,
Lax entropy condition, linear degeneracy
Integral curves, rarefaction waves
General solution of the Riemann Problem, shock collision

Week 9. Godunov's Method and High Order Methods (Ch. 13)
The Gourant-Isaacson-Rees method, Godnunov's method
Artificial viscosity, flux-limiter methods, slope-limiter methods

Week 10. Nonlinear Stability (Ch. 15-17)
Convergence, compactness, total variation stability, total variation diminishing methods,
monotonicity preserving methods, ENO scheme

Math 213C: Numerical Methods for Stochastic Partial Differential Equations
UC Riverside Spring 2022

Instructor: TBA (possible instructors: Weitao Chen, Yat Tin Chow, Heyrim Cho),
Office: TBA, **Email**: TBA
Lectures: TBA (3 hours per week), **Room**: TBA, **Office Hour**: TBA

Primary Textbook:
- Numerical Solution of Stochastic Differential Equations, by Peter E. Kloeden, Eckhard Platen, Springer Berlin Heidelberg, 2013
- Stochastic Finite Elements: A Spectral Approach, by Roger G Ghanem, Pol D Spanos, Courier Dover Publications, 2003.

Suggested References:
- Convex Optimization: Algorithms and Complexity, by Sébastien Bubeck, Foundations and Trends in Machine Learning, Vol. 8, No. 3-4, 2015.

Course Description: This course is an introduction to methods for computation of numerical solutions to stochastic differential equations and stochastic partial differential equations, as well as stochastic methods for optimization. This class is essential for students who would like to conduct research in scientific computing, large-scale optimization and numerical methods whenever stochastic processes are involved. It will teach students important and useful tools to compute numerical solutions to SDE and SPDE models describing different dynamical processes with applications in interdisciplinary research including fluid dynamics, mathematical biology, physics, engineering, chemistry and biology. The course will cover multiple approaches for different types of equations and study their properties including consistency, convergence and accuracy in the applications. The overall objective of this course is to provide students with the necessary mathematical knowledge and computational tools to understand and solve a variety of models in terms of stochastic differential equations, as well as equip the students with basic tools for stochastic optimization and machine learning.

Grading Criteria:
• Homework assignments (40%). Written homework problems, which will be assigned each week, are from the textbook. Homework problems include solving theoretical and practical problems inspired by the topics discussed in class. Extra reading assignments will be given weekly to read specific chapters including theories and examples in the primary textbook. The reading assignments will be evaluated by giving problems related to the materials in the written homework.
• Project assignments (40%). Computing projects will be assigned for each main topic, in addition to written homework, in which the numerical methods are studied in the context of a specific problem from scientific research. The computing projects will involve a

combination of numerical simulation and mathematical analysis.

• Final examination (20%), given during the final exam time, closed book written exam with about 6 problems.

Tentative Course Schedule
This outline covers 10 weeks of two 80-minute lectures each week.

Week 1. Introduction to stochastic ordinary differential equations (SDEs) (Book 1, Ch. 2-4)
Definition of stochastic process
Brownian motion, stochastic integrals (Results stated without proofs)

Week 2. Introduction to SDEs, numerical methods for SDEs (Book 1, Ch. 5)
Ito formula, Ornstein-Uhlenbeck process (Results stated without proofs)
Euler-Maruyama method

Week 3. Other numerical methods for SDEs (Book 1, Ch. 10)
Milstein's method, Metropolis-Adjusted Langevin Algorithm (MALA)

Week 4. Introduction to stochastic partial differential equations (SPDEs) (Book2, Ch. 2)
Karhunen-Loève (KL) expansion
PDEs with stochastic coefficients, PDEs with stochastic initial and boundary conditions

Week 5. Numerical methods for SPDEs (Book 2, Ch. 3)
Stochastic finite element method

Week 6. Numerical methods for SPDEs (Book 2, Ch. 3)
Stochastic spectral methods

Week 7. Other numerical methods for SPDEs with parameterized stochasticity (Book 2, Ch. 2-3)
General polynomial chaos, probabilistic collocation method

Week 8. Introduction to optimization and stochastic optimization (Book 3, Ch. 3)
Definition of convex optimization, gradient and subgradient
Gradient descent algorithms

Week 9. Stochastic methods for large scale optimization (Book 3, Ch. 6)
Stochastic gradient descent (SGD), comparison with Markov Chain Monte Carlo (MCMC)

Week 10. Other stochastic methods for large-scale optimization (Book 3, Ch. 6)
Mini-batch SGD, stochastic variance reduced gradient descent (SVRG)