Wave propagation problems arise in a wide range of applications. In this presentation, we will present and analyze finite element discontinuous Galerkin (DG) methods for two types of wave problems. In the first part of the talk, we consider the nonlinear shallow-water wave equations with a non-flat bottom topography, which have been widely used to model flows in rivers and coastal areas. Since the equations admit non-trivial steady-state solutions, extra care need to be taken to approximate the source term numerically. We will talk about recently developed high-order DG methods, which can capture the steady state well, and at the same time are positivity preserving without loss of mass conservation. In the second part, we will present energy conserving DG methods for the nonlinear dispersive Korteweg-de Vries (KdV) equation, which preserve the first two invariants (the integral and L2 norm) of the numerical approximations. Numerical results show that this property imparts the approximations with beneficial attributes such as more faithful reproduction of the amplitude and phase of traveling wave solutions. Extension to other wave propagation problems will be discussed at the end.