

# UNIVERSITY OF CALIFORNIA RIVERSIDE

#### DEPARTMENT OF MATHEMATICS

# COLLOQUIUM

### **David Carfi**

(Visiting Researcher, UCR – University of Messina)

"Extended Linear Algebra in Distributions Spaces and Dirac Calculus"

Abstract:

In his famous treatise "Principles of Quantum Mechanics", Paul Dirac introduced several "manipulation rules" for vectors and operators in a linear space, which together constitute the so-called "Dirac Calculus". This Calculus is nothing more than a wide set of formal extensions of the basic properties of the finite-dimensional Linear Algebra to the case of infinite-dimensional vector spaces. The discourse is elegant and surprisingly efficient, but it is far from being a rigorous mathematical analysis. As mathematicians well know, the passage from the finite to the infinite dimensional case does not amount to a mere substitution of finite linear combinations with formal integrals!

The goal of the research introduced in this talk is to give a precise mathematical meaning and rigorous support to many analytic methods of Quantum Mechanics, starting from Dirac Calculus, under as few general conditions as possible. This approach will give a rigorous justification for the use of these tools, leaving them substantially "as they are". Moreover, by providing a correct interpretation of these methods in terms of new mathematical entities and concepts, we will be helped in reaching a deeper understanding of the physical structures studied in Quantum Mechanics.



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The operation of continuous-superposition and that of the Dirac product allows us to build - in a mathematically rigorous way - the extended Linear Algebra of Dirac in the spaces of distributions, via their natural topologicallinear structures. More precisely, we shall see that the algebraic-topological structure of those spaces allows us to define naturally the linear combinations of a continuous family of vectors and operators and the scalar product of a vector by such families of vectors.

Moreover, we reread some classic theorems of Functional Analysis in terms of the extended linear algebra, which casts in a new light and perspective some fundamental chapters of the weak-duality theory developed by Jean Dieudonné and Laurent Schwartz between F and LF spaces.

> Wednesday, May 26<sup>th</sup>, 2010 Surge 284 4:10-5:00pm Tea Time at 3:40pm