Percy Deift conjectured that solutions to the KdV equation with almost periodic initial data exist globally and are almost periodic in time. Recent work by Ilia Binder, Michael Goldstein, Milivoje Lukic, and the speaker has established this conjecture under suitable assumptions on the spectral properties of the Schrödinger operator with potential given by the almost periodic initial condition in question. Specifically, it is assumed that the spectrum as a set is not too thin and the spectral measures are absolutely continuous. These properties have been verified for small analytic quasi-periodic functions with Diophantine frequency vector. This talk will present the history and context of the problem, and explain the BDGL approach to the Deift conjecture.