Syllabus for the qualifying examination in Analysis

Undergraduate materials:

- $\mathbb{R}$ and $\mathbb{R}^n$
- Basic topology: compact and connected sets, convergent sequences, Cauchy sequences, metric space completion.
- Sequences and series – numerical
- Continuity
- Differentiation
- Riemann integral
- Sequences and series of function, uniform convergence
- Fourier series (Chapter 8 from [8])
- Several variables: differentiation, inverse and implicit function theorem, stokes theorem
- Stone-Weierstrass theorem
- Arzela-Ascoli theorem

Measure Theory (209A)

- Properties of both abstract and Lebesgue-Stieltjes measures
- Caratheodory extension process constructing a measure on a sigma-algebra from a premeasure on an algebra; construction of Lebesgue-Stieltjes measure via this process
- Borel measures; complete measures; sigma-finite measure spaces
- Properties of measurable functions
- Abstract integration as well as Lebesgue integration on $\mathbb{R}^n$
- Dominated and monotone convergence theorems, Fatou’s Lemma
- Special examples: Cantor sets, Cantor function, construction of a non-Lebesgue measurable subset of $[0, 1]$. 

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• Modes of convergence: pointwise, uniform, almost everywhere, in measure, in $L^1$-norm, and implications between modes of convergence; Egoroff’s and Lusin’s theorems

• Product measures: Fubini-Tonelli’s theorem

• Relation of Lebesgue integral to Riemann integral

• Radon-Nikodym theorem; Hahn, Jordan, and Lebesgue decompositions

• Lebesgue’s differentiation theorem in $\mathbb{R}^n$; functions of bounded variation, absolute continuity

Functional analysis (209B)

• Normed vector spaces: Banach spaces, quotients, adjoints, Hahn-Banach Theorem, Baire Category theorem, open mapping theorem, closed graph theorem, the uniform boundedness principle.

• Topological vector spaces: weak topology, weak-* topology, Alaoglu’s theorem.

• Hilbert space: Schwartz’ inequality, Parallelogram law, Pythagorean theorem, Bessel’s inequality, Parseval’s identity

• $L^p$ spaces and $l^p$ spaces: Hölder, Minkowski inequalities, duals

Fourier analysis (209C)

• Various classes of functions: $C^\infty$, $C_c^\infty$, $C_c$, $C_0$, Schwarz class. Urysohn’s lemma.

• Convolution, Fourier transform and its properties. Fourier inversion theorem Young’s, Hausdorff-Young inequalities.

References


[2] W. Rudin, Real and Complex Analysis

[3] H. Royden, Real Analysis
[8] W. Rudin, Principles of Mathematical Analysis (undergraduate material)
[9] T. Apostol, Mathematical Analysis (undergraduate material)