Background material

1. "Undergraduate material" in the syllabi for real and complex analysis
2. Items 1–3 in the "undergraduate material" on groups in the syllabus for algebra
3. Items 1–2 in the "undergraduate material" on rings and fields in the syllabus for algebra
4. Items 1–4 in the "undergraduate material" on modules and linear algebra in the syllabus for algebra

4. Basics of set theory including the standard basic results on transfinite cardinal numbers (the Axiom of Choice will be used as needed, but Zorn’s Lemma and its applications are not required).

General Topology

1. Metric and topological spaces, open and closed sets
2. Continuous functions, homeomorphisms, related concepts
3. Constructions on spaces and mappings: Subspaces, products, quotients
4. Connectedness, local connectedness and path connectedness
5. Compactness (excluding Tychonoff’s Theorem)
6. Countability and separation properties, specializations to metric spaces
7. Local compactness, one point compactification
8. Complete metric spaces, completions (no existence proofs), Baire’s Theorem

Introduction to Algebraic Topology

1. Homotopy of continuous mappings, basic properties
2. Construction of the fundamental group, important general properties
3. Fundamental groups of important examples, applications
4. Covering spaces
5. The Seifert-van Kampen Theorem
6. Computational applications
7. Lifting criterion, existence and classification of coverings

Introduction to the theory of manifolds

1. Topological manifolds, partitions of unity
2. Local theory of smooth functions
3. Differential manifolds, global theory of smooth functions
4. Constructions on smooth manifolds
5. Tangent bundles, regular mappings, diffeomorphisms
6. Vector fields, integral curves, Lie brackets, completeness
7. Vector bundles, cross sections, constructions on vector bundles
8. Differential forms and exterior differential calculus (local theory)