Suppose that we have a graph $G$ where each vertex has $d$ edges and there are no cycles of length 4 or less. It is not hard to see that $G$ has at least $d^2+1$ vertices. The question is: can $G$ have exactly $d^2+1$ vertices? As it turns out, this is only possible if $d$ is one of the following numbers: 1, 2, 3, 7 or 57!

We'll see how this (surprisingly?) follows from the Spectral Theorem for symmetric matrices.